1. Problem 4.6, p 201, Johnson & Wichern (2007)
4. You are studying success in graduate school as measured by GPA. Using multiple regression, you want to predict GPA using a measure of the overall ability and a measure of the relative strength of verbal skills relative to quantitative skills. Since you have GRE scores available, you decide to form two “new” variables that are linear combinations of the verbal \(X_1\) and quantitative GRE \(X_2\) scores. As your measure of overall ability, you use the sum \((X_1 + X_2)\), and as your measure of relative strength of verbal skills, you use the difference \((X_1 - X_2)\). Your advisor is concerned that these two new variables would be highly correlated and lead to unstable estimates of your regression coefficients. Suppose that in the population of interest, the covariance matrix of verbal and quantitative scores, \(X = (X_1, X_2)'\), is 
\[
\Sigma = \begin{pmatrix} 100 & 50 \\ 50 & 100 \end{pmatrix}
\]
Is your advisor’s concern warranted? Explain your answer. Does it make sense?
5. Suppose that \(\hat{\beta}' = (\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\beta}_4)\) are estimates of regression coefficients and \(\beta \sim N(\beta, \Sigma)\). You want to test 
\[
H_0 : \beta_1 = \beta_2 = \beta_3
\]
(a) What linear combination(s) of \(\hat{\beta}\) would you need?
(b) What is the distribution for this linear combination(s)?

Hint: The \(H_0\) given above is equivalent to 
\[
H_0 : \beta_1 = \beta_2 \quad \text{and} \quad \beta_2 = \beta_3.
\]