
(a) Graphics: For the univariate distributions here are stem-n-leaf and boxplot. Alternatively you could have included histogram or dot diagram.

**Sales:**

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
<th>#</th>
<th>Boxplot</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>679</td>
<td>3</td>
<td>+-------+</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>57</td>
<td>2</td>
<td>+</td>
</tr>
<tr>
<td>1</td>
<td>01</td>
<td>2</td>
<td><em>-------</em></td>
</tr>
<tr>
<td>0</td>
<td>679</td>
<td>3</td>
<td>+-------+</td>
</tr>
</tbody>
</table>

---+---+---+---+

*Comment:* The sales distribution is “disjoint” and a bit U-shaped.

**Profits:**

<table>
<thead>
<tr>
<th>Stem</th>
<th>Leaf</th>
<th>#</th>
<th>Boxplot</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>3</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>5</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>60</td>
<td>2</td>
<td>+------+</td>
</tr>
<tr>
<td>14</td>
<td>17</td>
<td>2</td>
<td><em>-------</em></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>91</td>
<td>2</td>
<td>+------+</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

---+---+---+---+

*Comment:* The distribution of profits is pretty spread out, possibly uni-modal.

**Scatter Plot: Sales by Profits**
Comment: There appears to be a positive linear relationship between sales and profits.

(b) Descriptive statistics: These are all in billions of dollars.

<table>
<thead>
<tr>
<th>Sales</th>
<th>Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{x}_1 = 155.60$</td>
<td>$\bar{x}_2 = 14.70$</td>
</tr>
<tr>
<td>$s_{11} = 7476.45$</td>
<td>$s_{22} = 26.19$</td>
</tr>
<tr>
<td>$s_{12} = 303.62$</td>
<td>$r_{12} = .69$</td>
</tr>
</tbody>
</table>

Interpretation of $r_{12}$: $r_{12} = .69$ indicates that sales and profits are “moderately” linearly related. This confirms what we saw in the scatter plot from part (a).

“moderately” is my subjective impression/interpretation of the size of the correlation.

2. Vectors: lengths, angles and projections.

Let $x' = [6, 1, 3]$, $y' = [-1, 2, 1]$ and $z' = [1, 1, 1]$.

(a) Lengths of vectors:
   Length of $x = 6.78233$
   Length of $y = 2.4494897$
   Length of $z = 1.7320508$

(b) The angle between
   $x$ and $z$ is $31.65^\circ$.
   $x$ and $y$ is $93.45^\circ$.
   $y$ and $z$ is $61.87^\circ$.
(c) Projections of \( \mathbf{x} \) and \( \mathbf{y} \) onto \( \mathbf{z} \):

Projection of \( \mathbf{x} \) onto \( \mathbf{z} \) = \[
\begin{bmatrix}
3.33 \\
3.33 \\
3.33
\end{bmatrix}
\]

Projection of \( \mathbf{y} \) onto \( \mathbf{z} \) = \[
\begin{bmatrix}
0.67 \\
0.67 \\
0.67
\end{bmatrix}
\]

(d) Projection of

\[
\mathbf{x} \text{ on } \mathbf{y} = \begin{bmatrix} 0.167 \\ -0.333 \\ -0.167 \end{bmatrix}
\]

\[
\mathbf{y} \text{ on } \mathbf{x} = \begin{bmatrix} -0.130 \\ -0.022 \\ -0.065 \end{bmatrix}
\]

The projection of \( \mathbf{x} \) on \( \mathbf{y} \) and the projection of \( \mathbf{y} \) and \( \mathbf{x} \) are not the same because the direction of \( \mathbf{y} \) (which is the direction of the projection of \( \mathbf{x} \) on \( \mathbf{y} \)) is not the same as the direction of \( \mathbf{x} \) (which is the direction of \( \mathbf{y} \) on \( \mathbf{y} \)).

3. Various vector and matrix operations.

(a) \( \mathbf{1}' \mathbf{A} = (5, 3, 7) \)

(b) It is not possible to compute \( \mathbf{1A} \), because the matrices do not conform; that is, \( \mathbf{1} \) is \((3 \times 1)\) and \( \mathbf{A} \) is \((3 \times 3)\). The number of columns of \( \mathbf{1} \) are not equal to the number of rows of \( \mathbf{A} \).

(c) \( \mathbf{A1} = (11, -1, 5) \)

(d) Not possible (matrices do not conform); that is, \( \mathbf{A} \) is \((3 \times 3)\) and \( \mathbf{1}' \) is \((1 \times 3)\). The number of columns of \( \mathbf{A}_{3 \times 3} \) are not equal to the number of rows of \( \mathbf{1}_3 \).

(e) \[
2\mathbf{A} = \begin{bmatrix}
12 & 2 & 8 \\
-4 & 0 & 2 \\
2 & 4 & 4
\end{bmatrix}
\]

(f) \[
\mathbf{A} + \mathbf{B} = \begin{bmatrix}
8 & 1 & 6 \\
-5 & 1 & 2 \\
3 & -2 & 3
\end{bmatrix}
\]
(g) 

$$2A - 3B = \begin{pmatrix} 6 & 2 & 2 \\ 5 & -3 & -1 \\ -4 & 16 & 1 \end{pmatrix}$$

(h) \((2A)' - (3B)'\) is the same as part (g) but transposed; that is,

$$\begin{pmatrix} 6 & 5 & -4 \\ 2 & -3 & 16 \\ 2 & -1 & 1 \end{pmatrix}$$

(i) 

$$AB = \begin{pmatrix} 17 & -15 & 17 \\ -2 & -4 & -3 \\ 0 & -6 & 6 \end{pmatrix}$$

(j) 

$$BA = \begin{pmatrix} 14 & 6 & 12 \\ -19 & -1 & -9 \\ 21 & 4 & 6 \end{pmatrix}$$

which is not the same are \(AB\) from part (i). They’re the same order but they

are not equal.

(k) 

$$A'B' = \begin{pmatrix} 14 & -19 & 21 \\ 6 & -1 & 4 \\ 12 & -9 & 6 \end{pmatrix}$$

(l) 

\((BA)' = A'B'\) as they should; that is, the matrices from parts (i) and (k) are

equal.

(m) 

$$CB = \begin{pmatrix} 8 & 0 & 8 \\ -9 & 3 & 3 \\ 2 & -4 & 1 \end{pmatrix}$$
(n) \[ BC = \begin{pmatrix} 8 & 0 & 2 \\ -12 & 3 & 1 \\ 8 & -12 & 1 \end{pmatrix} \]

The matrices \( CB \neq BC \) because (in general) order in matrix multiplication matters. However note that the diagonals are the same. This results because matrix \( C \) is a diagonal.

In \( CB \), the rows of \( B \) are multiplied by the corresponding diagonals of \( C \). In \( BC \), the columns of \( B \) are multiplied by the corresponding diagonals of \( C \).

4. If \( A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \), \( B = \begin{pmatrix} -2 & 1 \\ 0 & 4 \end{pmatrix} \), and \( C = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} \) verify that

(a) \( (A + B) + C = A + (B + C) \)

\[ (A + B) + C = \begin{pmatrix} 0 & 5 \\ 1 & 7 \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 3 & 8 \end{pmatrix} \]

and

\[ A + (B + C) = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 3 & 6 \\ 3 & 8 \end{pmatrix} \]

(b) \( (AB)C = A(BC) \)

\[ (AB)C = \begin{pmatrix} -4 & 18 \\ -2 & 13 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 24 & 14 \\ 20 & 11 \end{pmatrix} \]

and

\[ A(BC) = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} -4 & -1 \\ 8 & 4 \end{pmatrix} = \begin{pmatrix} 24 & 14 \\ 20 & 11 \end{pmatrix} \]

(c) \( A(B + C) = AB + AC \)

\[ A(B + C) = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} 10 & 24 \\ 7 & 17 \end{pmatrix} \]

and

\[ AB + AC = \begin{pmatrix} -4 & 18 \\ -2 & 13 \end{pmatrix} + \begin{pmatrix} 14 & 6 \\ 9 & 4 \end{pmatrix} = \begin{pmatrix} 10 & 24 \\ 7 & 17 \end{pmatrix} \]